Varieties of Erotetic Implication

Andrzej Wiśniewski

Chair of Logic and Cognitive Science
Institute of Psychology
Adam Mickiewicz University
Poznań, Poland
Andrzej.Wisniewski@amu.edu.pl

Logic and Cognition
Poznań, May 17-19, 2012

Outline

What is Inferential Erotetic Logic?

- Inferential Erotetic Logic (IEL for short) is a logic that analyzes inferences in which questions perform the role of conclusions, and proposes criteria of validity for these inferences.
- The following semantical concepts are introduced:
  - *evocation* of questions by sets of declarative sentences/d-wffs, and
  - *erotetic implication* of questions by questions and sets of declarative sentences/d-wffs.
- **Validity** is then defined in terms of evocation or erotetic implication, depending on the type of inference under consideration.
- The general setting of IEL does not require the underlying logic of declaratives to be classical. In other words, IEL is neutral in the controversy concerning what "The Logic" of declaratives is.

What is Inferential Erotetic Logic?

- Inferential Erotetic Logic (IEL for short) is a logic that analyzes inferences in which questions perform the role of conclusions, and proposes criteria of validity for these inferences.
- The following semantical concepts are introduced:
  - ***evocation*** of questions by sets of declarative sentences/d-wffs, and
  - ***erotetic implication*** of questions by questions and sets of declarative sentences/d-wffs.
- **Validity** is then defined in terms of evocation or erotetic implication, depending on the type of inference under consideration.
- The general setting of IEL does not require the underlying logic of declaratives to be classical. In other words, IEL is neutral in the controversy concerning what "The Logic" of declaratives is.

What is Inferential Erotetic Logic?

- Inferential Erotetic Logic (IEL for short) is a logic that analyzes inferences in which questions perform the role of conclusions, and proposes criteria of validity for these inferences.
- The following semantical concepts are introduced:
  - ***evocation*** of questions by sets of declarative sentences/d-wffs, and
  - ***erotetic implication*** of questions by questions and sets of declarative sentences/d-wffs.
- **Validity** is then defined in terms of evocation or erotetic implication, depending on the type of inference under consideration.
- The general setting of IEL does not require the underlying logic of declaratives to be classical. In other words, IEL is neutral in the controversy concerning what "The Logic" of declaratives is.

What is Inferential Erotetic Logic?

- Inferential Erotetic Logic (IEL for short) is a logic that analyzes inferences in which questions perform the role of conclusions, and proposes criteria of validity for these inferences.
- The following semantical concepts are introduced:
  - ***evocation*** of questions by sets of declarative sentences/d-wffs, and
  - ***erotetic implication*** of questions by questions and sets of declarative sentences/d-wffs.
- **Validity** is then defined in terms of evocation or erotetic implication, depending on the type of inference under consideration.
- The general setting of IEL does not require the underlying logic of declaratives to be classical. In other words, IEL is neutral in the controversy concerning what "The Logic" of declaratives is.

What is Inferential Erotetic Logic?

- Inferential Erotetic Logic (IEL for short) is a logic that analyzes inferences in which questions perform the role of conclusions, and proposes criteria of validity for these inferences.
- The following semantical concepts are introduced:
  - ***evocation*** of questions by sets of declarative sentences/d-wffs, and
  - ***erotetic implication*** of questions by questions and sets of declarative sentences/d-wffs.
- **Validity** is then defined in terms of evocation or erotetic implication, depending on the type of inference under consideration.
- The general setting of IEL does not require the underlying logic of declaratives to be classical. In other words, IEL is neutral in the controversy concerning what "The Logic" of declaratives is.

What is Inferential Erotetic Logic?

- Inferential Erotetic Logic (IEL for short) is a logic that analyzes inferences in which questions perform the role of conclusions, and proposes criteria of validity for these inferences.
- The following semantical concepts are introduced:
  - ***evocation*** of questions by sets of declarative sentences/d-wffs, and
  - ***erotetic implication*** of questions by questions and sets of declarative sentences/d-wffs.
- **Validity** is then defined in terms of evocation or erotetic implication, depending on the type of inference under consideration.
- The general setting of IEL does not require the underlying logic of declaratives to be classical. In other words, IEL is neutral in the controversy concerning what “The Logic” of declaratives is.

Examples of Erotetic Implication

Is Andrew lying?
Andrew lies if, and only if he speaks very slowly.

Does Andrew speak very slowly?

Where did Andrew leave for: Paris, London or Moscow?
If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.
If Andrew departed in the morning, then he left for Paris or London.
If Andrew departed in the evening, then he left for Moscow.

When did Andrew depart: in the morning, or in the evening?

Where does Andrew live?
Andrew lives in a university town in Western Poland.

Which towns in Western Poland are university towns?

Examples of Erotetic Implication

Is Andrew lying?
Andrew lies if, and only if he speaks very slowly.

Does Andrew speak very slowly?

Where did Andrew leave for: Paris, London or Moscow?
If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.
If Andrew departed in the morning, then he left for Paris or London.
If Andrew departed in the evening, then he left for Moscow.

When did Andrew depart: in the morning, or in the evening?

Where does Andrew live?
Andrew lives in a university town in Western Poland.

Which towns in Western Poland are university towns?

Examples of Erotetic Implication

Is Andrew lying?
Andrew lies if, and only if he speaks very slowly.

Does Andrew speak very slowly?

Where did Andrew leave for: Paris, London or Moscow?
If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.
If Andrew departed in the morning, then he left for Paris or London.
If Andrew departed in the evening, then he left for Moscow.

When did Andrew depart: in the morning, or in the evening?

Where does Andrew live?
Andrew lives in a university town in Western Poland.

Which towns in Western Poland are university towns?

Examples of Erotetic Implication

Is Andrew lying?
Andrew lies if, and only if he speaks very slowly.

Does Andrew speak very slowly?

Where did Andrew leave for: Paris, London or Moscow?
If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.
If Andrew departed in the morning, then he left for Paris or London.
If Andrew departed in the evening, then he left for Moscow.

When did Andrew depart: in the morning, or in the evening?

Where does Andrew live?
Andrew lives in a university town in Western Poland.

Which towns in Western Poland are university towns?

Examples of Erotetic Implication

Is Andrew lying?
Andrew lies if, and only if he speaks very slowly.

Does Andrew speak very slowly?

Where did Andrew leave for: Paris, London or Moscow?
If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.
If Andrew departed in the morning, then he left for Paris or London.
If Andrew departed in the evening, then he left for Moscow.

When did Andrew depart: in the morning, or in the evening?

Where does Andrew live?
Andrew lives in a university town in Western Poland.

Which towns in Western Poland are university towns?

Examples of Erotetic Implication

Is Andrew lying?
Andrew lies if, and only if he speaks very slowly.

Does Andrew speak very slowly?

Where did Andrew leave for: Paris, London or Moscow?
If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.
If Andrew departed in the morning, then he left for Paris or London.
If Andrew departed in the evening, then he left for Moscow.

When did Andrew depart: in the morning, or in the evening?

Where does Andrew live?
Andrew lives in a university town in Western Poland.

Which towns in Western Poland are university towns?

Soundness of a question

- **Soundness**: A question is *sound* iff at least one direct answer to the question is true.
- Some questions are sound, but some other are not!

Soundness of a question

- **Soundness**: A question is *sound* iff at least one direct answer to the question is true.
- Some questions are sound, but some other are not!

Erotetic Implication: Intuitions

Roughly, question Q (erotetically) ***implies*** question Q_1 ***on the basis of*** a set X of declarative sentences/formulas iff:

(I) (TRANSMISSION OF SOUNDNESS/TRUTH INTO SOUNDNESS):

- *If Q is sound and X consists of truths, then Q_1 must be sound.*

(II) (OPEN-MINDED COGNITIVE USEFULNESS):

- *For each direct answer B to Q_1 there exists a non-empty proper subset Y of the set of direct answers to Q such that the following condition holds:*
  (•) if B is true and X consists of truths, then at least one direct answer (to Q) in Y must be true.

Erotetic Implication: Intuitions

Roughly, question Q (erotetically) ***implies*** question Q_1 ***on the basis of*** a set X of declarative sentences/formulas iff:

(I) (TRANSMISSION OF SOUNDNESS/TRUTH INTO SOUNDNESS):

- *If Q is sound and X consists of truths, then Q_1 must be sound.*

(II) (OPEN-MINDED COGNITIVE USEFULNESS):

- *For each direct answer B to Q_1 there exists a non-empty proper subset Y of the set of direct answers to Q such that the following condition holds:*
  (•) if B is true and X consists of truths, then at least one direct answer (to Q) in Y must be true.

Erotetic Implication: Intuitions

Roughly, question Q (erotetically) ***implies*** question Q_1 ***on the basis of*** a set X of declarative sentences/formulas iff:

(I) (TRANSMISSION OF SOUNDNESS/TRUTH INTO SOUNDNESS):

- *If Q is sound and X consists of truths, then Q_1 must be sound.*

(II) (OPEN-MINDED COGNITIVE USEFULNESS):

- *For each direct answer B to Q_1 there exists a non-empty proper subset Y of the set of direct answers to Q such that the following condition holds:*
  (•) if B is true and X consists of truths, then at least one direct answer (to Q) in Y must be true.

Erotetic Implication: Intuitions

Roughly, question Q (erotetically) ***implies*** question Q_1 ***on the basis of*** a set X of declarative sentences/formulas iff:

(I) (TRANSMISSION OF SOUNDNESS/TRUTH INTO SOUNDNESS):

- *If Q is sound and X consists of truths, then Q_1 must be sound.*

(II) (OPEN-MINDED COGNITIVE USEFULNESS):

- *For each direct answer B to Q_1 there exists a non-empty proper subset Y of the set of direct answers to Q such that the following condition holds:*
  (•) if B is true and X consists of truths, then at least one direct answer (to Q) in Y must be true.

Erotetic Implication: Intuitions

Roughly, question Q (erotetically) ***implies*** question Q_1 ***on the basis of*** a set X of declarative sentences/formulas iff:

(I) (TRANSMISSION OF SOUNDNESS/TRUTH INTO SOUNDNESS):

- *If Q is sound and X consists of truths, then Q_1 must be sound.*

(II) (OPEN-MINDED COGNITIVE USEFULNESS):

- *For each direct answer B to Q_1 there exists a non-empty proper subset Y of the set of direct answers to Q such that the following condition holds*:
  (•) *if B is true and X consists of truths, then at least one direct answer (to Q) in Y must be true.*

Erotetic Implication: Intuitions

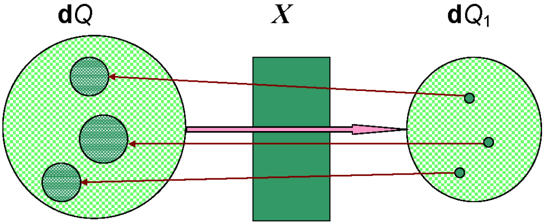

Figure: *Two directions*

Examples of Erotetic Implication

Is Andrew lying?
Andrew lies if, and only if he speaks very slowly.
Does Andrew speak very slowly?

Where did Andrew leave for: Paris, London or Moscow?
If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.
If Andrew departed in the morning, then he left for Paris or London.
If Andrew departed in the evening, then he left for Moscow.
When did Andrew depart: in the morning, or in the evening?

Where does Andrew live?
Andrew lives in a university town in Western Poland.
Which towns in Western Poland are university towns?

Did Andrew fly by BA, or by Rynair, or by neither of them?
Did Andrew fly by BA?

Examples of Erotetic Implication

Is Andrew lying?
Andrew lies if, and only if he speaks very slowly.
Does Andrew speak very slowly?

Where did Andrew leave for: Paris, London or Moscow?
If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.
If Andrew departed in the morning, then he left for Paris or London.
If Andrew departed in the evening, then he left for Moscow.
When did Andrew depart: in the morning, or in the evening?

Where does Andrew live?
Andrew lives in a university town in Western Poland.
Which towns in Western Poland are university towns?

Did Andrew fly by BA, or by Rynair, or by neither of them?
Did Andrew fly by BA?

Examples of Erotetic Implication

Is Andrew lying?
Andrew lies if, and only if he speaks very slowly.
Does Andrew speak very slowly?

Where did Andrew leave for: Paris, London or Moscow?
If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.
If Andrew departed in the morning, then he left for Paris or London.
If Andrew departed in the evening, then he left for Moscow.
When did Andrew depart: in the morning, or in the evening?

Where does Andrew live?
Andrew lives in a university town in Western Poland.
Which towns in Western Poland are university towns?

Did Andrew fly by BA, or by Rynair, or by neither of them?
Did Andrew fly by BA?

Examples of Erotetic Implication

Is Andrew lying?
Andrew lies if, and only if he speaks very slowly.
Does Andrew speak very slowly?

Where did Andrew leave for: Paris, London or Moscow?
If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.
If Andrew departed in the morning, then he left for Paris or London.
If Andrew departed in the evening, then he left for Moscow.
When did Andrew depart: in the morning, or in the evening?

Where does Andrew live?
Andrew lives in a university town in Western Poland.
Which towns in Western Poland are university towns?

Did Andrew fly by BA, or by Rynair, or by neither of them?
Did Andrew fly by BA?

Definition of Erotetic Implication

Definition

(EROTETIC IMPLICATION) $\mathbf{Im}(Q, X, Q_1)$ iff:

1. for each $A \in \mathbf{d}Q : X \cup \{A\} \| \models \mathbf{d}Q_1$, and
2. for each $B \in \mathbf{d}Q_1$ there exists a non-empty proper subset Y of $\mathbf{d}Q$ such that $X \cup \{B\} \| \models Y$.

Remarks:

$\mathbf{d}Q$ and $\mathbf{d}Q_1$ stand for the sets of *direct answers* to Q and Q_1, respectively.

It is assumed that a question has at least two (sentential) direct answers.

$\| \models$ denotes *multiple-conclusion entailment*.

A comparison: Belnap's interrogative entailment

- Belnap assigns the logical values, Truth and Falsity, to questions. A question Q is said to be *true in a model* $\boldsymbol{M}$ if at least one direct answer to Q is true in $\boldsymbol{M}$ (for simplicity, we disregard here Belnap's distinction between interrogatives and questions). Let us use the term *quasiformulas* as a cover term for declarative formulas and questions of a formal language.
- A straightforward generalization of the standard concept of entailment is:

(♡) *a set of quasiformulas* Ψ *entails a quasiformula* γ *iff* γ *is true in each model in which all the quasiformulas in* Ψ *are true.*

A comparison: Belnap's interrogative entailment

- Belnap assigns the logical values, Truth and Falsity, to questions. A question Q is said to be *true in a model* $\boldsymbol{M}$ if at least one direct answer to Q is true in $\boldsymbol{M}$ (for simplicity, we disregard here Belnap's distinction between interrogatives and questions). Let us use the term *quasiformulas* as a cover term for declarative formulas and questions of a formal language.
- A straightforward generalization of the standard concept of entailment is:

(♡) *a set of quasiformulas* Ψ *entails a quasiformula* γ *iff* γ *is true in each model in which all the quasiformulas in* Ψ *are true.*

A comparison: Belnap's interrogative entailment

- Belnap assigns the logical values, Truth and Falsity, to questions. A question Q is said to be *true in a model* $\boldsymbol{M}$ if at least one direct answer to Q is true in $\boldsymbol{M}$ (for simplicity, we disregard here Belnap's distinction between interrogatives and questions). Let us use the term *quasiformulas* as a cover term for declarative formulas and questions of a formal language.
- A straightforward generalization of the standard concept of entailment is:

(♡) *a set of quasiformulas* Ψ *entails a quasiformula* γ *iff* γ *is true in each model in which all the quasiformulas in* Ψ *are true.*

A comparison: Belnap's interrogative entailment

- As a special case we get:

($\heartsuit^{\star}$) *a question Q_1 is entailed by a question Q together with a set of declarative formulas X iff $X \cup \{Q\}$ entails Q_1.*

- Given some obvious assumptions, Belnap-style entailment of Q_1 from $X \cup \{Q\}$ holds just in case the first clause of the definition of erotetic implication is fulfilled.
- But recall that erotetic implication is defined by means of *two* clauses.

A comparison: Belnap's interrogative entailment

- As a special case we get:

($\heartsuit^{\star}$) *a question Q_1 is entailed by a question Q together with a set of declarative formulas X iff $X \cup \{Q\}$ entails Q_1.*

- Given some obvious assumptions, Belnap-style entailment of Q_1 from $X \cup \{Q\}$ holds just in case the first clause of the definition of erotetic implication is fulfilled.
- But recall that erotetic implication is defined by means of *two* clauses.

A comparison: Belnap's interrogative entailment

- As a special case we get:

($\heartsuit^{\star}$) *a question Q_1 is entailed by a question Q together with a set of declarative formulas X iff $X \cup \{Q\}$ entails Q_1.*

- Given some obvious assumptions, Belnap-style entailment of Q_1 from $X \cup \{Q\}$ holds just in case the first clause of the definition of erotetic implication is fulfilled.
- But recall that erotetic implication is defined by means of *two* clauses.

A comparison: Belnap's interrogative entailment

- As a special case we get:

($\heartsuit^\star$) *a question Q_1 is entailed by a question Q together with a set of declarative formulas X iff $X \cup \{Q\}$ entails Q_1.*

- Given some obvious assumptions, Belnap-style entailment of Q_1 from $X \cup \{Q\}$ holds just in case the first clause of the definition of erotetic implication is fulfilled.
- But recall that erotetic implication is defined by means of *two* clauses.

A comparison: Groenendijk and Stokhof's interrogative entailment

Let us now consider the proposal present in Groenendijk and Stokhof [1996]. The underlying idea is:

> *"(...) interrogatives* $?\phi_1 \ldots ?\phi_n$ *entail (...) interrogative* $?\psi$ *in a model* ***M*** *iff any proposition which completely answers all of the* $?\phi_1 \ldots ?\phi_n$ *in* ***M****, also completely answers* $?\psi$ *in* ***M****. Logical entailment amounts to entailment in all models." (Groenendijk and Stokhof [1996], p. 1090).*

A comparison: G & S interrogative entailment

- Thus as a special case we get something like:

(♠) *a question Q entails a question Q_1 iff for each model **M**, any proposition that completely answers Q in **M**, completely answers Q_1 in **M** as well.*

- The idea resembles that of *containment* in the sense of Hamblin [1958]: a question Q contains a question Q_1 iff from each answer to Q one can deduce some answer to Q_1.

A comparison: G & S interrogative entailment

- Thus as a special case we get something like:

(♠) *a question Q entails a question Q_1 iff for each model **M**, any proposition that completely answers Q in **M**, completely answers Q_1 in **M** as well.*

- The idea resembles that of *containment* in the sense of Hamblin [1958]: a question Q contains a question Q_1 iff from each answer to Q one can deduce some answer to Q_1.

A comparison: G & S interrogative entailment

- Thus as a special case we get something like:

(♠) *a question Q entails a question Q_1 iff for each model* ***M****, any proposition that completely answers Q in* ***M****, completely answers Q_1 in* ***M*** *as well.*

- The idea resembles that of *containment* in the sense of Hamblin [1958]: a question Q contains a question Q_1 iff from each answer to Q one can deduce some answer to Q_1.

A comparison: G & S interrogative entailment

The second clause of the definition of erotetic implication expresses an idea similar to that of Groenendijk and Stokhof. Yet, the original idea is generalized, since partial answers are allowed and declarative premises are taken into consideration. One can easily prove:

Corollary

Let $B \in \mathbf{d}Q_1$. If $X \cup \{B\}$ entails some direct or partial answer to Q, then there exists a non-empty proper subset Y of $\mathbf{d}Q$ such that $X \cup \{B\} \mid\!\models Y$.

A comparison: G & S interrogative entailment

Partial answers are defined by:

Definition

A declarative formula A is a partial answer to a question Q iff $A \notin [\mathbf{d}Q]$, but for some non-empty proper subset Y of $\mathbf{d}Q$:

(1) $A \,||\!\!\models Y$, and

(2) for each $B \in Y$: $B \models A$.

A comparison: G & S interrogative entailment

- The converse holds assuming that mc-entailment is compact and a certain additional condition is true.
- The relation $||\!\!=$ of mc-entailment is said to be *compact* if whenever $X \,||\!\!= Y$, then $X_1 \,||\!\!= Y_1$ for some finite subsets X_1 of X, and Y_1 of Y.

A comparison: G & S interrogative entailment

- The converse holds assuming that mc-entailment is compact and a certain additional condition is true.
- The relation $||\!\!\models$ of mc-entailment is said to be *compact* if whenever $X \;||\!\!\models Y$, then $X_1 \;||\!\!\models Y_1$ for some finite subsets X_1 of X, and Y_1 of Y.

A comparison: G & S interrogative entailment

We have:

Corollary

Let $B \in \mathbf{d}Q_1$. Suppose that there exists a non-empty proper subset Y of $\mathbf{d}Q$ such that $X \cup \{B\} \,||\!\!= Y$. Suppose further that mc-entailment in the language is compact, the language includes disjunction $\vee$, and the following condition holds:

($\triangledown$) *for each admissible partition $\mathbf{P} = \langle \mathbf{T_P}, \mathbf{U_P} \rangle$ of the language:*

$$\{A_1, \ldots, A_n\} \cap \mathbf{T_P} \neq \emptyset \text{ iff } \ulcorner A_1 \vee \ldots \vee A_n \urcorner \in \mathbf{T_P}.$$

Then $X \cup \{B\}$ entails a direct or partial answer to Q.

A comparison: G & S interrogative entailment

- The Groenendijk-Stokhof analysis does not provide, however, any counterpart of the first clause of the definition of erotetic implication.

From implying question to implied question

It is assumed that Q (erotetically) implies Q_1 on the basis of X, and that $\mathbf{P} = \langle \mathbf{T_P}, \mathbf{U_P} \rangle$ is an arbitrary but fixed admissible partition of the relevant language.

Table: *From implying question to implied question*

Q	X	Q_1
sound in **P**	$X \subset \mathbf{T_P}$	*sound in* **P**
unsound in **P**	$X \subset \mathbf{T_P}$	*unsound in* **P**
sound in **P**	$X \not\subset \mathbf{T_P}$	*sound in* **P** *or unsound in* **P**
unsound in **P**	$X \not\subset \mathbf{T_P}$	*sound in* **P** *or unsound in* **P**

From implied question to implying question

Table: *From implied question to implying question*

Q_1	X	Q
sound in **P**	$X \subset \mathbf{T_P}$	*sound in* **P**
unsound in **P**	$X \subset \mathbf{T_P}$	*unsound in* **P**
sound in **P**	$X \not\subset \mathbf{T_P}$	*sound in* **P** *or unsound in* **P**
unsound in **P**	$X \not\subset \mathbf{T_P}$	*sound in* **P** *or unsound in* **P**

Narrowing down vs. elimination

Let us compare the conditions:

(nd) *$B \in \mathbf{d}Q_1$ and there exists a non-empty proper subset Y of $\mathbf{d}Q$ such that $X \cup \{B\} \mid\models Y$.*

(el) *$B \in \mathbf{d}Q_1$ eliminates on the basis of X a certain direct answer C to Q.*

where "B eliminates C on the basis of X" means "for each admissible partition $\mathbf{P} = \langle \mathbf{T_P}, \mathbf{U_P} \rangle$: if $X \cup \{B\} \subset \mathbf{T_P}$, then $C \in \mathbf{U_P}$".

They are not equivalent!

Narrowing down vs. elimination

- Let Q = '? $\{p, q\}$', $X = \emptyset$, and Q_1 = '? p'. Thus $\mathbf{d}Q = \{p, q\}$ and $\mathbf{d}Q_1 = \{p, \neg p\}$. Clearly, $\neg p$ eliminates p. But $\neg p$ entails neither p nor q, and hence $\neg p$ does not mc-entail any proper subset of $\{p, q\}$.
- Let Q = '? $\{q, r, s\}$', $X = \{p \to q, \neg p \to r \vee s\}$, and Q_1 = '? p'. Now $\neg p$ mc-entails, together with X, the proper subset $\{r, s\}$ of $\mathbf{d}Q$. However, $\neg p$ does not eliminate, on the basis of X, any direct answer to Q. When one gets $\neg p$, it is still possible that q holds.

Narrowing down vs. elimination

- Let Q = '? $\{p, q\}$', $X = \emptyset$, and Q_1 = '? p'. Thus $\mathbf{d}Q = \{p, q\}$ and $\mathbf{d}Q_1 = \{p, \neg p\}$. Clearly, $\neg p$ eliminates p. But $\neg p$ entails neither p nor q, and hence $\neg p$ does not mc-entail any proper subset of $\{p, q\}$.
- Let Q = '? $\{q, r, s\}$', $X = \{p \rightarrow q, \neg p \rightarrow r \vee s\}$, and Q_1 = '? p'. Now $\neg p$ mc-entails, together with X, the proper subset $\{r, s\}$ of $\mathbf{d}Q$. However, $\neg p$ does not eliminate, on the basis of X, any direct answer to Q. When one gets $\neg p$, it is still possible that q holds.

Narrowing down vs. elimination

However, the following is true:

Corollary

If condition (**el**) *is fulfilled and, in addition, the following condition holds:*

$$(\mathbf{rs}) \quad X \Vdash \mathbf{d}Q$$

then condition (**nd**) *is fulfilled as well.*

Narrowing down vs. elimination

We also have:

Corollary

If condition (**nd**) *is fulfilled and the following condition holds:*

(**me**) *for any* $A, C \in \mathbf{d}Q$*, where* $A \neq C$*:*

(•) *A eliminates C*

then condition (**el**) *is fulfilled as well.*

General vs. existential

The second clause of the definition of erotetic implication, i.e.

- *for each* $B \in \mathbf{d}Q_1$ *there exists a non-empty proper subset* Y *of* $\mathbf{d}Q$ *such that* $X \cup \{B\} \,||\!\!\models Y$.

requires each direct answer to an implied question to be cognitively useful wrt the implying question. One can claim that this is too much: only some ("most expected"?) answers should do. Thus one can put:

- *there exists* $B \in \mathbf{d}Q_1$ *such that for some non-empty proper subset* Y *of* $\mathbf{d}Q : X \cup \{B\} \,||\!\!\models Y$.

However, the concept obtained that way is too general to give an intuitive account of validity of erotetic inferences (of the second kind).

General vs. existential

Another variant of "existential" erotetic implication is this:

Definition (*Eliminative erotetic implication*)

A question Q eliminatively implies a question Q_1 on the basis of a set of d-wffs X (in symbols: $\mathbf{Im_G}(Q, X, Q_1)$) iff:

(1) for each $A \in \mathbf{d}Q : X \cup \{A\} \mid\models \mathbf{d}Q_1$, and

(2) for some $B \in \mathbf{d}Q_1$: B eliminates, on the basis of X, a certain direct answer to Q.

Elimination yields narrowing down on the condition:

- $X \mid\models \mathbf{d}Q$

which, by the way, is trivially satisfied when direct answers are supposed to cover the whole "logical space". However, IEL does not presuppose this.

Regular erotetic implication

Definition (*Regular erotetic implication*)

A question Q regularly implies a question Q_1 on the basis of a set of d-wffs X iff

(1) for each $A \in \mathbf{d}Q : X \cup \{A\} \mid\models \mathbf{d}Q_1$, and

(2) for each $B \in \mathbf{d}Q_1$ there exists $C \in \mathbf{d}Q$ such that $X \cup \{B\} \models C$.

Strong erotetic implication

Definition (*Strong erotetic implication*)

A question Q strongly implies a question Q_1 on the basis of a set of d-wffs X iff

(1) for each $A \in \mathbf{d}Q : X \cup \{A\} \Vdash \mathbf{d}Q_1$, and

(2) for each $B \in \mathbf{d}Q_1$ there exists a non-empty proper subset Y of $\mathbf{d}Q$ such that $X \cup \{B\} \Vdash Y$, but $X \nVdash Y$.

Pure erotetic implication

Definition (*Pure erotetic implication*)

A question Q implies a question Q_1 (in symbols: $\mathbf{Im}(Q, Q_1)$) iff:

(1) for each $A \in \mathbf{d}Q : A \mid\!\models \mathbf{d}Q_1$, and

(2) for each $B \in \mathbf{d}Q_1$ there exists a non-empty proper subset Y of $\mathbf{d}Q$ such that $B \mid\!\models Y$.

Analytic erotetic implication

Definition (*Analytic erotetic implication*)

A question Q analytically implies a question Q_1 iff $\mathbf{Im}(Q, Q_1)$ and each immediate subformula of a direct answer to Q_1 is a subformula of a direct answer to Q or is a negation of a subformula of a direct answer to Q.